

ANALYTICAL FORM OF THE DEUTERON WAVE FUNCTION CALCULATED WITHIN THE DISPERSION APPROACH

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We present a convenient analytical parametrization of the deuteron wave function calculated within dispersion approach as a discrete superposition of Yukawa-type functions, in both configuration and momentum spaces.

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Recently in the paper [1] it was shown that the deuteron tensor polarization component $T_{20}(Q^2)$ provides a crucial test of deuteron wave functions in the range of momentum transfers available in to-day experiments. The calculation [1] shows that the most popular model wave functions do not give adequate description of $T_{20}(Q^2)$ and are to be discriminate in favor of those obtained in the dispersion potentialless inverse scattering approach with no adjustable parameters [2], (see [3], too) and giving the best description. Some time ago this function (MT - wave function) was used in the calculation of the neutron charge form factor [4]. The results of calculation (12 new points) are compatible with existing values of this form factor of other authors. A fit is obtained for the whole set (36 points) taking into account the data for the slope of the form factor at $Q^2 = 0$. These results will be used in the neutrino scattering experiments in Fermilab [5].

The aim of the present paper is to present a conventional algebraic parametrization of the deuteron MT-wave function calculated within dispersion approach as a discrete superposition of Yukawa-type terms.

Let us remind briefly the main characteristic features of these wave functions obtained in the frame of the potentialless approach to the inverse scattering problem.

The important feature of these wave functions is the fact that they are "almost model independent": no form of NN interaction Hamiltonian is used. The MT wave functions are given by the dispersion type integral directly in terms of the experimental scattering phases and the mixing parameter for NN scattering in the 3S_1 - 3D_1 channel. Regge-analysis of experimental data on NN scattering was used to describe the phase shifts at large energy.

It is worth to notice that the MT wave functions were

obtained using quite general assumptions about analytical properties of quantum amplitudes such as the validity of the Mandelstam representation for the deuteron electrodisintegration amplitude. These wave functions have no fitting parameters and can be altered only with the amelioration of the NN scattering phase analysis.

Let us notice that the process of constructing of these wave functions is closely related to the equations obtained in the framework of the dispersion approach based on the analytic properties of the scattering amplitudes [6, 7] (see also [8] and especially the detailed version [9]). In fact, this approach is a kind of dispersion technique using integrals over composite-system masses.

Let us emphasize that by construction the dispersion wave functions [2] differ principally from deuteron wave functions used in the conventional nuclear model (see, e.g., the review [10]). So, for applications it is convenient to have MT-wave functions [2] in analytical form.

Therefore, we present here a simple parametrization of the deuteron function as a superposition of Yukawa-type terms (that was introduced in Ref. [11] for Paris potential; see also the fit in Ref. [12] for CD-Bonn wave function).

So, we consider the deuteron wave functions $\varphi_l(r)$ in the states with orbital momentum $l = 0$, $\varphi_0(r) = u(r)$ and $l = 2$, $\varphi_2(r) = w(r)$. The ansatz for the analytic versions of the r -space wave functions, denoted by $u_a(r)$ and $w_a(r)$, is

$$u_a(r) = \sum_{j=1}^{n_u} C_j \exp(-m_j r),$$

$$w_a(r) = \sum_{j=1}^{n_w} D_j \exp(-m_j r) \left[1 + \frac{3}{m_j r} + \frac{3}{(m_j r)^2} \right], \quad (1)$$

$$m_j = \alpha + m_0(j-1),$$

where the coefficients C_j , D_j , the maximal value of the index j and m_0 are defined by the condition of the best

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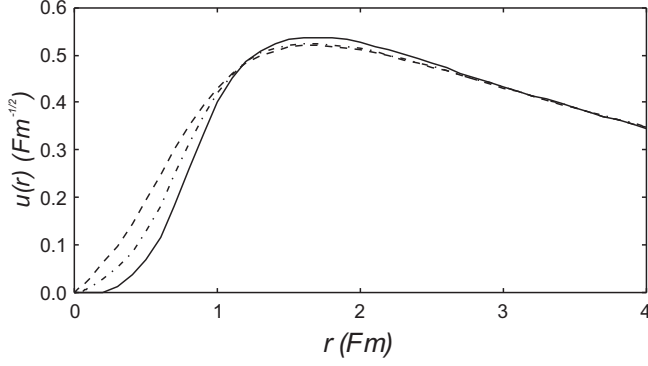


FIG. 1: S-wave deuteron wave function. Solid line - MT [2, 3], dashed - [12], dot-dashed - [11].

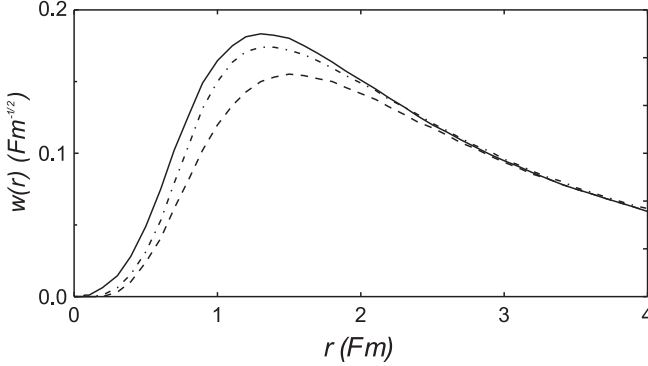


FIG. 2: D-wave deuteron wave function. Legend is the same as in Fig.1.

fitting. $\alpha = \sqrt{M \varepsilon_d}$, M is nucleon mass, ε_d is the binding energy of deuteron.

These wave functions are normalized according to

$$\int_0^\infty dr \left[(u(r))^2 + (w(r))^2 \right] = 1. \quad (2)$$

The conventional boundary conditions at zero:

$$u(r) \sim r, \quad w(r) \sim r^3, \quad (3)$$

lead to one condition for C_j and three constraints for D_j , namely:

$$\sum_{j=1}^{n_u} C_j = 0, \quad \sum_{j=1}^{n_w} D_j = \sum_{j=1}^{n_w} D_j m_j^2 = \sum_{j=1}^{n_w} \frac{D_j}{m_j^2} = 0. \quad (4)$$

Using the form (1) it is easy to describe the standard behaviour of the deuteron wave functions at $r \rightarrow \infty$. The asymptotics of S -state is

$$u(r) \sim A_S e^{-\alpha r}, \quad (5)$$

and the asymptotics of D state is

$$w(r) \sim \eta A_S \left(1 + \frac{3}{\alpha r} + \frac{3}{(\alpha r)^2} \right) e^{-\alpha r}. \quad (6)$$

TABLE I: Coefficients for the parametrized deuteron wave function calculated within a dispersion approach. The last C_j and last three D_j are to be computed from eq. (4) ($n_u=16$, $n_w=12$).

C_j (fm $^{-1/2}$)	D_j (fm $^{-1/2}$)
0.87872995+00	0.22245143-01
-0.50381047+00	-0.41548258+00
0.28787196+02	-0.18618515+01
-0.82301294+03	0.21987598+02
0.12062383+05	-0.16885413+03
-0.10574260+06	0.76001430+03
0.59534957+06	-0.22287203+04
-0.22627706+07	0.43330023+04
0.59953379+07	-0.54072021+04
-0.11282284+08	eq. (4)
0.15181681+08	eq. (4)
-0.14519973+08	eq. (4)
0.96491938+07	
-0.42403857+07	
0.11092702+07	
eq. (4)	

Here A_S and $A_D = \eta A_S$ are the asymptotic S -state and D -state normalizations and η is the asymptotic “ D/S state ratio”. In our calculation of MT-wave-function we use $\alpha=0.231625$ fm $^{-1}$.

The Fourier-transforms of wave functions $\psi_l(k)$, $l = 0, 2$ in the momentum representation in r -space are:

$$\frac{\varphi_l(r)}{r} = \sqrt{\frac{2}{\pi}} \int_0^\infty k^2 dk j_l(kr) \psi_l(k), \quad (7)$$

where $j_l(kr)$ is the spherical Bessel function.

The normalization condition for these wave functions is given by

$$\int_0^\infty k^2 dk \left[(\psi_0(k))^2 + (\psi_2(k))^2 \right] = 1. \quad (8)$$

The fits for momentum space wave functions, following from (1) and (7) are

$$\begin{aligned} \psi_0^a(k) &= \sqrt{\frac{2}{\pi}} \sum_j \frac{C_j}{(k^2 + m_j^2)}, \\ \psi_2^a(k) &= \sqrt{\frac{2}{\pi}} \sum_j \frac{D_j}{(k^2 + m_j^2)}. \end{aligned} \quad (9)$$

The calculated coefficients in the fits (1), (9) are listed in the table 1.

The asymptotics at $r \rightarrow \infty$ gives for the obtained fits of MT-wave functions the asymptotic “ D/S state ratio”

$$\eta = \frac{D_1}{C_1} = 0.02531511. \quad (10)$$

The accuracy of this parametrization is illustrated by the magnitudes of the integrals:

$$\left\{ \int_0^\infty dr [u(r) - u_a(r)]^2 \right\}^{1/2} = 4.1 \cdot 10^{-3}, \quad (11)$$

$$\left\{ \int_0^\infty dr [w(r) - w_a(r)]^2 \right\}^{1/2} = 2.2 \cdot 10^{-3} . \quad (12)$$

So, we present a convenient analytical parametrization of the deuteron wave function calculated within disper-

sion approach as a discrete superposition of Yukawa-type functions. This function is plotted on the Fig.1 and Fig.2. The wave functions [11] and [12] are given for comparison.

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